1. Introduction

Distributed modeling of catchment hydrology became an attractive approach for the analysis and forecasting of the transformation of rainfall into runoff. However, limited knowledge of model inputs (initial and boundary conditions, parameters) and observations of the hydrological response make the underlying problems of calibration, sensitivity analysis and uncertainty analysis very challenging. Besides, rainfall-runoff is a typical case where the dimension of the system response to be analyzed (or cost function to be optimized) is small compared to the number of input parameters to be prescribed. In this case, the adjoint technique is very efficient in computing the gradient of an objective function w.r.t. all parameters. In this prospective study, the approach is applied to the infiltration and overland flow components of a distributed flash flood model. Using synthetic experiments, the potential of variational methods for model analysis, multi-criteria and multi-variable sensitivity analysis as well as parameter estimation is illustrated.

2. Adjoint method

The adjoint technique is a very convenient way to obtain the derivatives of a scalar function w.r.t. with respect to a large parameter space because computational cost is independent from its dimension.

Objective function

\[ J(x, a, i) = \int \psi(x, a, i) \, dt \]

Forward model

\[ \frac{\partial x}{\partial t} + \frac{\partial (xu)}{\partial x} = \frac{q}{\partial x} \]

Adjoint model

\[ \frac{\partial p}{\partial t} + \frac{\partial (pu)}{\partial x} = \frac{q}{\partial x} \]

\[ \nabla_w J = \begin{bmatrix} \frac{\partial J}{\partial x_1} & \cdots & \frac{\partial J}{\partial x_n} \end{bmatrix} \]

The adjoint model is used in the variational data assimilation framework to compute the gradient of a misfit cost function between model results and observations. This gradient is used in a limited quasi-Newton optimization algorithm to find the control vector (parameters to be estimated) leading to the lower value for the cost function. Given the results of the preliminary sensitivity analysis and the range or parameters variations, K and n are catchment key parameters to be estimated.

\[ K = \sum n_k V_k \]

Cost Function

\[ J_r = \frac{1}{2} \left( \int \psi(x, a, i) \, dt - \int \psi_d(x, a, i) \, dt \right)^2 \]

Convergence for \( K_{ini} = 10 \) and \( K_{ini} > 10 \)

3. Flash flood model

The underlying physics of MARINE, a distributed and event based flash flood model developed at IMFT (Espana et al., 2000), is adapted to events for which infiltration excess dominates the generation of the flood (partial area infiltration excess overland flow).

\[ \frac{\partial h}{\partial t} + \frac{\partial (h^2)}{\partial x} = \frac{q}{\partial x} \]

\[ \psi = \frac{h}{\psi} \]

\[ \psi = \max \left( 0, \frac{h}{\psi} \right) \]

\[ \psi = \max \left( 0, \frac{h}{\psi} \right) + \sqrt{\psi^2 + q^2} \]

4. Adjoint sensitivity analysis

The adjoint model is a very effective alternative to compute the gradient of a response w.r.t. parameters. Appropriate measures of the model response were chosen and the adjoint model yields to the sensitivity to input parameters.

Runoff coefficient

\[ y_1 = \int_0^t q(x, a, i) \, dt \]

Composite response

\[ y_2 = \frac{\max (y_1)}{\min (y_2)} \]

Global analysis

After normalization and aggregation of the sensitivities at the catchment scale, relative parameters contributions to the hydrologic responses can be compared.

5. Variational data assimilation

This prospective study illustrated the potential of adjoint sensitivity analysis and variational data assimilation for catchment scale hydrology applications. The deterministic sensitivity analysis is only local but provided interesting information along segments of the parameter space. The spatial and temporal patterns can be used for model parameterization, observations weighting and model coupling. The use of the singular value decomposition should be investigated for rigorous analysis of parameters identifiability.

Parameters estimation trough data assimilation is possible but require appropriate regularization approaches and strategies for the reduction of the control space to be developed. Such investigations are in progress and should lead to a well posed inverse problem.

Lastly, when observation data is assimilated to the model, the model is no longer the dynamic model itself but the optimality system with forward model, adjoint model, cost function and optimality condition. Therefore, second order adjoint is necessary to take in account the uncertainty related to observations.

References


Acknowledgements

This work is supported by a bi-annual research contract between the Midi-Pyrénées region and the French government funded by the Midi-Pyrénées region and the French government funded by the Midi-Pyrénées region.